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Determination of the angle of rotation of the diffraction grating by the method of conical diffraction

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Diffractograms of rotated polyaniline fiber grating were studied experimentally. Based on the obtained results, the diffraction patterns were analyzed and set of experimental data obtained after image digitization was approximated by the least squares method. The angle of rotation of the diffraction grating was calculated using the determined coefficients of the second-order curves.

Key words: Conical diffraction, diffraction grating, optical system, laser.

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Introduction

Off-plane diffraction of light (or conical diffraction CD) causes the distortion of the diffraction pattern (diffracted rays spread along to the surface of a cone) when the laser beam is obliquely incident on the diffraction grating (DG) [1–7]. The position of the individual diffraction maxima (or a band formed by intersection of the diffraction maxima) in this case will be in general described by a second order curve on the screen projection of the resulting diffractogram. The shape of this curve (e.g., hyperbola, ellipse or parabola) depends on the orientation of the grating in space (or on the angular position of the laser if the position of the grating is fixed). This effect is observed for both transmission and reflection DG, in particular, for phase gratings with a spatially modulated refractive index (see e.g., [4]). Furthermore, the problem of rays diffraction for arbitrary space orientation of DG is important not only for visible radiation, but, for example, in the design of X-ray spectrometers [8], etc.

An important and promising application of the CD effect is the construction of technologically simple and low-priced sensors, which can be used to determine the spatial position of the source of optical radiation [9]. The polyaniline fibers DG (refractive index $n \simeq 1.5$) can be easily integrated into various materials. The CD effects will be observed when the laser radiation incident obliquely on such system and, as a result, the geometric parameters (i.e., curve shape, curvature, etc.) of the diffraction curves can be easily determined using regression methods (see e.g., [10–12, 14]).

The purpose of this article is to determine the angular position of the polyaniline fibers DG based on the analysis of the shape and geometric parameters of the diffraction curves.

I. Experimental Details

The scheme of the experiment is depicted in Fig. 1. A grating consisting of two layers of polyaniline fiber (the diameter of fiber and the distance between the grating layers was $\simeq 0.16$, and 1 mm) as well as a source of light (He-Ne laser with wavelength $\lambda = 632.8$ nm) formed an optical system. The DG was placed on a rotary table with the angle measuring scale (the accuracy was no worth than ±1).

The screen projections of diffraction patterns were recorded using a Canon 80d digital camera (CMOS-matrix with ~24 MP resolution).

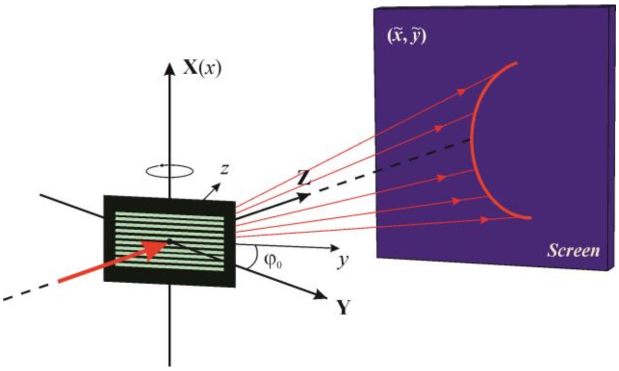


Fig. 1. The scheme of the experiment.

Diffraction curves were stored for each fixed position of the DG. The rotation angle of the grating φ_0 was varied with a step of 5° from 5° to 45°, respectively. The obtained images were digitized applying the trial version of GetData Graph Digitizer 2.26 program.

The images of the diffraction curves obtained for various grating orientation angles are presented in the Fig. 2. In addition, these curves have hyperbolic shape and this feature is typical for the given orientation angles of the diffraction element (see e.g., [9]).

A general approximation method of data which are described by a second-order curve (2.1) was used in our investigation.

$$a\tilde{x}^2 + b\tilde{x}\tilde{y} + c\tilde{y}^2 + d\tilde{x} + e\tilde{y} + f = 0. \qquad (1.1)$$

An unknown a, b, c, d, e, f coefficients were determined using a Python program (see e.g., the code of the program in [13]) based on algorithm [10].

II. Results and Discussion

Let us consider the problem of the light beam propagation at an angle θ_0 relatively to the normal of the DG (in the plane perpendicular to the grating plane and perpendicular to the direction of the grooves). In this case the position of the diffraction maxima is determined by the well known equation [15, 16]:

$$\sin\theta_0 \mp \sin\theta_m = \mp\frac{m\lambda}{d}, \quad m = 0, \pm1, \pm2, \ldots \qquad (2.1)$$

where θ_m denotes the angle at which the m order diffraction maximum is observed, λ is the wavelength of light, d is the grating period. The sign "+" corresponds to the reflection and "–" to the transmission diffraction grating, respectively.

In contrast, the diffractogram will have the form of a second-order curve when a laser beam is obliquely incident on a DG. The diffraction equations in the direction cosines approximation for an arbitrary space orientation of the grating are expressed as follows [1–3]:

$$\alpha_i \mp \alpha_m = \mp\frac{m\lambda}{d}\sin\psi, \quad \beta_i \mp \beta_m = \mp\frac{m\lambda}{d}\cos\psi, \qquad (2.2)$$

where the index i corresponds to the incident beam with the direction cosines which are described by the following equations:

$$\alpha_i = -\sin\theta_0\cos\varphi_0, \quad \beta_i = -\sin\varphi_0. \qquad (2.3)$$

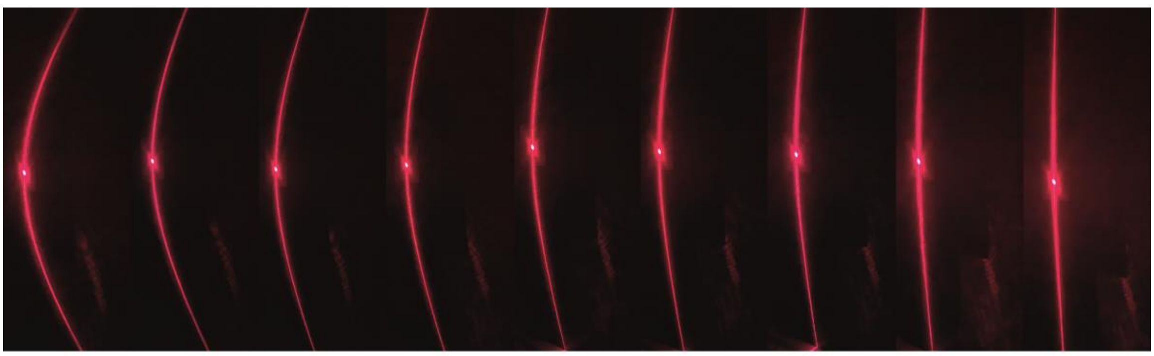
In addition, both α_m and β_m expressed in terms of the coordinates of the mth order diffraction maximum for the diffracted beam are as follows:

$$\alpha_m = \frac{x_m}{\sqrt{x_m^2+y_m^2+z_m^2}}, \quad \beta_m = \frac{y_m}{\sqrt{x_m^2+y_m^2+z_m^2}}. \qquad (2.4)$$

The ψ is the angle between the direction of the grooves and the X axis, φ_0 corresponds to the angle of DG rotation (the angle between the beam propagation direction i.e. Z axis and the z axis, see Fig. 1). In addition, there are $\psi = \pi/2$ and $\theta_0 = 0$ for our experiment. As a result, substituting both (3.3) and (3.4) into the equation (3.2) we obtained the following formula:

$$x_m^2 + z_m^2 - y_m^2\cot^2\varphi_0 = 0. \qquad (2.5)$$

This equation describes a cone with axis along y direction (parallel to the grating grooves) and with a circular section with radius: $R = y_m\cot\varphi_0$. Furthermore, the cross-section of the diffraction cone (3.5) will be described by a second-order curve [9] on the plane of the screen placed perpendicular to the Z axis at a distance of l:



φ_0=45° φ_0=40° φ_0=35° φ_0=30° φ_0=25° φ_0=20° φ_0=15° φ_0=10° φ_0=5°

Fig. 2. The diffraction patterns obtained for φ_0 = 5°…45°, orientations of DG.

$$x_m^2 \cos^4 \varphi + z_m^2 \cos 2\varphi + 2 z_m l \sin^3 \varphi - l^2 \sin^2 \varphi = 0. \qquad (2.6)$$

where $\varphi = \pi/2 - \varphi_0$.

As can be seen from the equation (3.6) there are the ellipse curves with the diffraction maxima for angles $\varphi < \pi/4$. In addition, there are a hyperbola and a parabola observed for $\varphi > \pi/4$ and $\varphi = \pi/4$, respectively. It is necessary to note, that there is a possibility to transform the discrete coordinates of maxima $\{x_m, z_m\}$ in to continuous $\{x, z\}$ ones in the equation (3.6) taking into account the diffraction phenomena on individual fibers and unfocusing effects (polymer fiber can be considered as a cylindrical lens). In addition, there is a sufficiently large period of our DG (the linear distance between the diffraction maxima on the screen is inversely proportional to the grating period [1]). Therefore, the diffractograms have the form of solid curves, and there are no separated diffraction maxima on the screen in our experiment. It is necessary to note, that the position of the main diffraction maximum for different orientation of the grating is clearly seen on the Fig. 2 (the brightest point).

The equation that can be used for the analysis of the experimental results (in the screen ($\tilde{x}$, $\tilde{y}$) coordinate system) is expressed as follows (the transformation $x_m \to \tilde{x}$, $z_m \to \tilde{y}/\cos\varphi$ of the coordinate system was applied):

$$\tilde{x}^2 \cos^4 \varphi + \tilde{y}^2 \frac{\cos 2\varphi}{\cos^2 \varphi} + 2\tilde{y} l \frac{\sin^2 \varphi}{\cos \varphi} - l^2 \sin^2 \varphi = 0. \quad (3.7)$$

The results of the digitized data for all range of the change of φ_0 grating rotation angle are presented in Fig. 3, 4 and 5, respectively.

As a matter of fact, the coefficients of the equation (2.1) could be multiplied by an arbitrary parameter (the same curve is observed), therefore, the normalized coefficients (e.g., c/a, etc) were used for further data analysis. An additional coefficient γ in the equation (3.7) was entered (by replacing the $\tilde{y} \to \gamma\tilde{y}$) to match the geometric parameters of projected images on the screen and digitized curves. Thus, there are two parameters l and γ, the value of which can be chosen to best agree the curves constructed by the equation (3.7) with the experimental data. Finally, to determine the unknown angle φ, one can use the equation: $c/a = \gamma^2 \cos 2\varphi / \cos^6 \varphi$.

The obtained curves (two applied parameters were used in our case:$\gamma = 1.2 \pm 0.4$ and $l = 0.6 \pm 0.2$) are presented in Fig. 3, 4 and 5. The values of the angle $\varphi_{0C} = f(\varphi_0)$ determined according to this method are presented on the Fig. 6 (the best fit to these data are presented as line with intercept parameter – 15° ± 6° and slope ones – 0.6 ± 0.2). As can be seen from this figure the φ_{0C} are calculated with significant errors (especially in the range from 5° to 20°) with respect to ideal dependence (see red lined on the Fig. 6). These particular results are due to insufficiently good agreement between the experimental data and the calculated curves (using equation 3.7) with given l and γ coefficients. In addition these peculiarities may be attributed to the influence of little change in the distance between the digital camera and the screen, as well as focusing (to obtain an optimal image of the diffraction pattern on the screen) during the experiment.

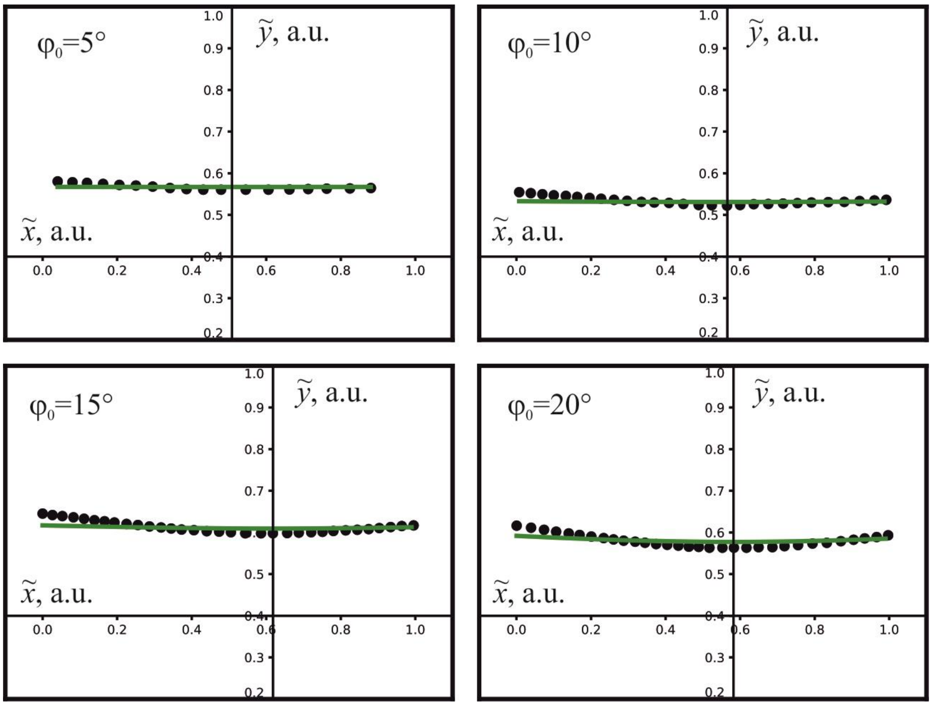


Fig. 3. The diffraction curves for φ_0 = 5°, 10°, 15°, 20° orientation angles of the diffraction grating. ● – digitized values; the curves built using the equation (3.7) are colored in green.

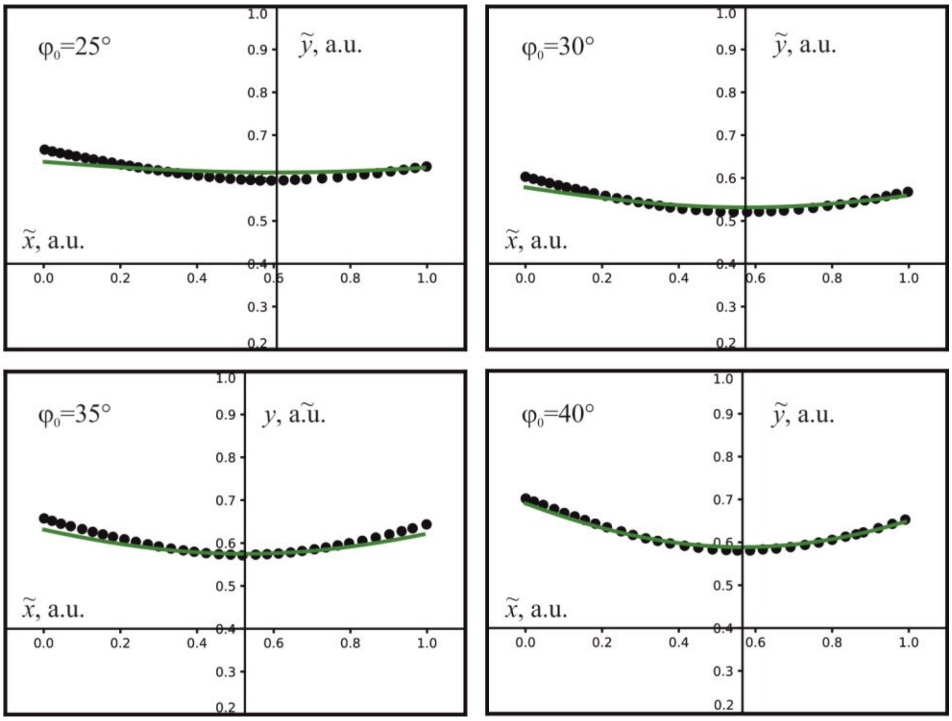


Fig. 4. The diffraction curves for φ_0 = 25°, 30°, 35°, 40° orientation angles of the diffraction grating. ● – digitized values; the curves built using the equation (3.7) are colored in green.

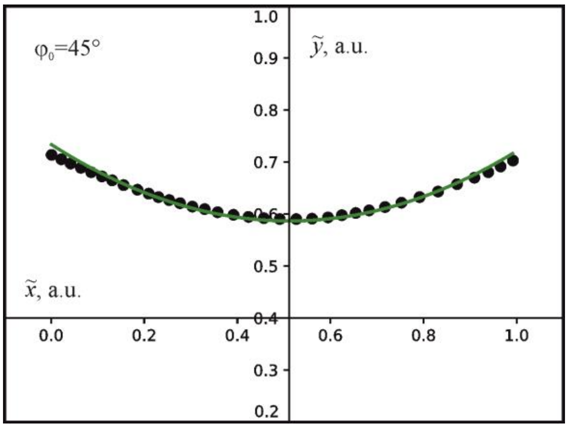


Fig. 5. The diffraction pattern for φ_0 = 45°orientation angle of the diffraction grating. ● – digitized values; the curves built using the equation (3.7) are colored in green.

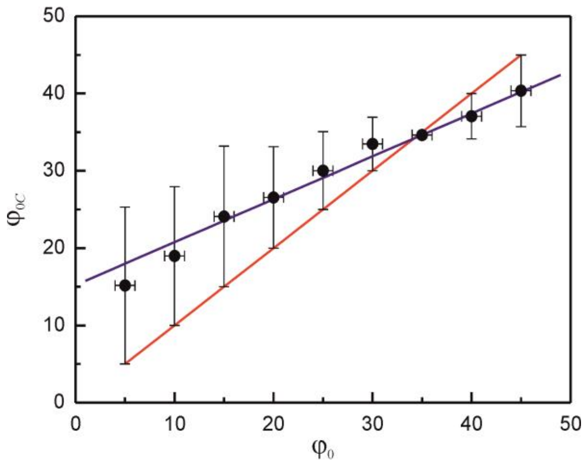


Fig. 6. The dependence of the calculated φ_{0C} angel of rotation of DG with respect to experimental set. Red line – ideal dependence, blue line – best fit of the calculated data.

Conclusions

The diffraction patterns were obtained from polyaniline fiber grating in the case of an oblique incidence of the laser beam. The main equations which describe the conical diffraction were applied to determine values of the coefficients *a*, *b*, *c*, *d*, *e*, *f* of the second order hyperbolic shape curves. The values of the grating rotation angles in the range from 5° to 45° were determined.

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Визначення кута повороту дифракційної ґратки методом конічної дифракції

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Отримано дифрактограми розвернутої ґратки, яка складається з волокон поліаніліну. Проведено аналіз оцифрованих дифракційних картин та апроксимовано методом найменших квадратів набір експериментальних даних. Розраховано кут повороту дифракційної ґратки на основі коефіцієнтів кривих другого порядку.

Ключові слова: Конічна дифракція, дифракційна ґратка, оптична система, лазер.