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HOMOMORPHISMS AND FUNCTIONAL CALCULUS IN ALGEBRAS OF ENTIRE FUNCTIONS ON BANACH SPACES

In the paper the homomorphisms of algebras of entire functions on Banach spaces to a commutative Banach algebra are studied. In particular, it is proposed a method of constructing of homomorphisms vanishing on homogeneous polynomials of degree less or equal than a fixed number n.

Key words and phrases: Aron-Berner extension, functional calculus, algebras of analytic functions on Banach spaces.

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1 INTRODUCTION AND PRELIMINARIES

In 1951 R. Arens [1] found a way of extending the product of Banach algebra A to its bidual A'' in such a way that this bidual became itself a Banach algebra. There are two canonical ways to extend the product from A to A'' which called the Arens products. We recall definitions [2].

Let A be a commutative Banach algebra, X be a Banach space over the field of complex numbers $\mathbb{C}$.

If $x \in X$ and $\lambda \in X'$ then we write $\langle \lambda, x\rangle = \lambda(x)$. For every $a, b \in A, \lambda \in A'$ and $\Phi \in A''$ define $a.\lambda \in A', \lambda.a \in A', \lambda.\Phi \in A'$ and $\Phi.\lambda \in A'$ by:

$$a.\lambda : b \mapsto \langle \lambda, ba\rangle, \lambda.a : b \mapsto \langle \lambda, ab\rangle,$$
$$\lambda.\Phi : b \mapsto \langle \Phi, b.\lambda\rangle, \Phi.\lambda : b \mapsto \langle \Phi, \lambda.b\rangle;$$

and then define two products $\Box$ and $\Diamond$ on A'' by:

$$\langle \Phi\Box\Psi, \lambda\rangle = \langle \Phi, \Psi.\lambda\rangle, \langle \Phi\Diamond\Psi, \lambda\rangle = \langle \Psi, \lambda.\Phi\rangle (\Phi, \Psi \in A'').$$

Then $(A'', \Box)$ and $(A'', \Diamond)$ are Banach algebras. We say that A is *Arens regular* if for all $\Phi, \Psi \in A''$ we have $\Phi\Box\Psi = \Phi\Diamond\Psi$.

For a given complex Banach space X, $\mathcal{P}(^nX)$ denotes the Banach space of all continuous n-homogeneous complex-valued polynomials on X. The problem of extending every element of $\mathcal{P}(^nX)$ to a continuous n-homogeneous polynomial $\widetilde{P}$ on the bidual X'' of X was first studied by Aron and Berner in 1978, who showed that such extensions always exist.

Let $B : X \times \ldots \times X \to \mathbb{C}$ be the symmetric n-linear mapping associated to P. B can be extended to an n-linear mapping $\widetilde{B} : X'' \times \ldots \times X'' \to \mathbb{C}$. Let $(z_1, \ldots, z_n) \in X'' \times \ldots \times X''$.

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For a net (x_{α_k}) from X which converges to z_k in the weak-star topology of X'' for each fixed $k, 1 \leqslant k \leqslant n$, we put

$$\widetilde{B}(z_1, \ldots, z_n) = \lim_{\alpha_1} \ldots \lim_{\alpha_n} B(x_{\alpha_1}, \ldots, x_{\alpha_n}).$$

Then *the Aron-Berner extension* P on X'' to X is defined as

$$\widetilde{P}(z) = \widetilde{B}(z, \ldots, z),$$

where B is a unique continuous n-linear symmetric form for which $P(x) = B(x, \ldots, x)$ for each $x \in X$.

Consider the complete projective tensor product $A \otimes_\pi X$. Every element of $A \otimes_\pi X$ can be represented by the form $\overline{a} = \sum_k a_k \otimes_\pi x_k$, where $a_k \in A,\ x_k \in X$. For every $\overline{a} \in A \otimes_\pi X$ and $f \in H_b(X)$ (algebra of entire analytic functions of bounded type on a Banach space X) let us define $\overline{f}(\overline{a})$ in the means of functional calculus for analytic functions on a Banach spaces ([5]). Then $\widetilde{\overline{f}}$ is the Aron-Berner extension of $\overline{f}$.

In [6] using the Aron-Berner extension and approach developed in [4] it was obtained a method to construct nontrivial complex homomorphisms of $H_b(X)$ vanishing on homogeneous polynomials of degree less or equal that a fixed number n. In this paper we extend this result for Banach algebra valued homomorphism.

2 MAIN RESULTS

Recall that X is *a left A-module* (X is *a left module over* A), if exists a bilinear map $A \times X \to X$, $(a, x) \mapsto a \cdot x$ such that $(a_1 \cdot a_2) \cdot x = a_1 \cdot (a_2 \cdot x)$, where $a_1, a_2 \in A, x \in X$. It is easy to prove that $A \otimes_\pi X$ is a left A-module. So, using Theorem 2 ([3], p.297) we can easy obtain the following proposition.

Proposition 1. *$(A \otimes_\pi X)''$ is a left A''-module.*

In [7] it is proved a theorem about a homomorphism of algebras $H_b(X)$ and $H_b((A \otimes_\pi X''), A)$ in the case when A is some finite dimensional algebra with identity. The following theorem extends this result for the case of an infinite dimensional algebra A.

Proposition 2. *Let A be the Arens regular Banach algebra. For every $f \in H_b(X)$ there exists a function $\widetilde{\overline{f}} \in H_b((A \otimes_\pi X)'', A'')$ such that $\widetilde{\overline{f}}(e \otimes x) = ef(x), x \in X$ and the mapping $F : f \mapsto \widetilde{\overline{f}}$ is a homomorphism between algebras $H_b(X)$ and $H_b((A \otimes_\pi X)'', A'')$.*

The proof it easy follows from the fact that both the Aron-Berner extension and functional calculus are topological homomorphisms ([4], [5]).

Example 1. *Let us show that in the case if A is not Arens regular, then the map F is not necessary a homomorphism. Let $A = \ell_1$, $X = \mathbb{C}^2$. We need to prof that*

$$F : H_b(\mathbb{C}^2) \to H_b((\ell_1 \otimes_\pi \mathbb{C}^2)'', \ell_1'') \text{ the are } f, g \in H_b(\mathbb{C}^2) \text{ such that } F(fg) \neq F(f)F(g).$$

For each $t = (t_1, t_2) \in \mathbb{C}^2$ put $f(t) = t_1, g(t) = t_2$ and apply the extension operator $\mathbb{C}^2 \ni t \rightsquigarrow x \in \ell_1 \times \ell_1$ and the Aron-Berner extension $\ell_1 \times \ell_1 \ni x \rightsquigarrow u = (u_1, u_2) \in \ell_\infty \times \ell_\infty$. Then

$$\overline{f}(x) = x_1 \in \ell_1, \quad \overline{g}(x) = x_2 \in \ell_1, \quad \overline{f}(x)\overline{g}(x) = x_1 * x_2,$$

where $''*''$ is the convolution product in ℓ_1. Suppose that

$$\widetilde{\widetilde{f}}(u) = u_1 \in \ell_1'', \qquad \widetilde{\widetilde{g}}(u) = u_2 \in \ell_1''.$$

Then we have $\widetilde{\widetilde{f}}(u)\widetilde{\widetilde{g}}(u) = u_1 \Box u_2$ and $\widetilde{\widetilde{g}}(u)\widetilde{\widetilde{f}}(u) = u_1 \Diamond u_2 = u_1 \Box u_2$.

Since $u_1 \Diamond u_2 \neq u_1 \Box u_2$ in the general case so, we can conclude that F is not a homomorphism.

On the other hand, $fg(t) = t_1 \cdot t_2 = P(t)$ — homogeneous polynomial of second degree vector variable t. It is known that $P(t) = B(t,t)$ is bilinear form which is uniquely determined by the polarization formula:

$$B(t,t) = \frac{t_1 t_2 + t_2 t_1}{2}.$$

Then

$$\overline{B}(x,x) = \frac{x_1 * x_2 + x_2 * x_1}{2},$$

and we have

$$\widetilde{\widetilde{B}}(u) = \frac{u_1 \Box u_2 + u_1 \Diamond u_2}{2} = \frac{u_2 \Box u_1 + u_2 \Diamond u_1}{2}.$$

So, $\widetilde{\widetilde{B}}(u,u) = \widetilde{\widetilde{P}}(u) = \widetilde{\widetilde{fg}}(t) \neq \widetilde{\widetilde{f}}(t)\widetilde{\widetilde{g}}(t)$.

Next, we consider the case when A is a reflexive Banach algebra. Let us denote by $\mathcal{P}(^nX)$ the Banach space of all continuous n-homogeneous complex-valued polynomials on X. $\mathcal{P}_f(^nX)$ denotes the subspace of n-homogeneous polynomials of finite type, that is, the subspace generated by finite sum of finite products of linear continuous functionals. The closure of $\mathcal{P}_f(^nX)$ in the topology of uniform convergence on bounded sets is called the space of approximable polynomials and denoted by $\mathcal{P}_c(^nX)$.

Let us denote by $A_n(X)$ the closure of the algebra, generated by polynomials from $\mathcal{P}(^{\leqslant n}X)$ with respect to the uniform topology on bounded subsets of X. It is clear that $A_1(X) \cap \mathcal{P}(^nX) = \mathcal{P}_c(^nX)$.

Let us denote by $\mathcal{L}(H_b(X), A)$ the space of all continuous n-linear operators on $H_b(X)$ to A and let $M_A(H_b(X))$ be the set of all homomorphisms on $H_b(X)$ to A.

In [4] introduced a concept of radius function $R(\varphi)$ of a given linear functional $\varphi \in H_b(X)'$ as the infimum of all numbers $r > 0$ such that φ is bounded with respect to the norm of uniform convergence on the ball rB and proved that

$$R(\varphi) = \limsup_{n\to\infty} \|\varphi_n\|^{1/n},$$

where φ_n is the restriction of φ to $\mathcal{P}(^nX)$. In [7] extended this definition to a homomorphism $\Phi \in M_A(H_b(X))$, that is, $R(\Phi)$ is the infimum of all numbers $r > 0$ such that Φ is bounded with respect to the norm of uniform convergence on the ball rB and proved that

$$R(\Phi) = \limsup_{n\to\infty} \|\Phi_n\|^{1/n}, \tag{1}$$

where Φ_n is the restriction of Φ to space n-homogeneous polynomials.

Theorem 1. *Suppose that $\Phi_n \in \mathcal{L}(\mathcal{P}(^nX), A)$ for $n \in \mathbb{Z}_+$, and suppose that the norms of Φ_n on $\mathcal{P}(^nX)$ satisfy*

$$\|\Phi_n\| \leq cs^n$$

for $c, s > 0$. Then there is a unique $\Phi \in \mathcal{L}(H_b(X), A)$ whose restriction to $\mathcal{P}(^nX)$ coincides with Φ_n for every $n \in \mathbb{Z}_+$.

Proof. For any character $\theta \in M(A), \|\theta\| = 1$ we construct operator $\Phi_n : \mathcal{P}(^n X) \to A$. Then $\theta \circ \Phi_n \in (\mathcal{P}(^n X))'$ and $\|\theta \circ \Phi_n\| \leq \|\Phi_n\|$. Since $\|\Phi_n\| \leq cs^n$, then every θ satisfies the inequality $\|\theta \circ \Phi_n\| \leq cs^n$. From [4, Proposition 2.4] it follows that for every θ there exists linear functional $\varphi : H_b(X) \to \mathbb{C}, \varphi \in H_b(X)'$, such that $\varphi_n = \theta \circ \Phi_n$. Therefore, we have operator $T : A' \to H_b(X)', \theta \mapsto \varphi$ and $T^\star$ is the adjoint operator to T:

$$T^\star : H_b(X)'' \to A'' = A.$$

Let us consider the restriction of $T^\star$ on $H_b(X) \subset H_b(X)''$ and denoted it by Φ. Clearly $\Phi : H_b(X) \to A$ is a required operator.

In order to prove that the restriction Φ to $\mathcal{P}(^n X)$ coincides with Φ_n it is enough to show that $\Phi_n(P) = \Phi(P)$ for every $P \in \mathcal{P}(^n X)$. Put $\Phi_n(P) = a_1$, $\theta(a_1) = c_1 \in \mathbb{C}$, that is $(\theta \circ \Phi_n)(P) = \varphi_n(P) = c_1$. On the other hand, $\Phi(P) = a_2$, that is $(\theta \circ \Phi)(P) = \varphi(P) = c_2$. Since φ_n is restriction of φ, $\varphi(P) = c_2 = \varphi_n(P) = c_1, \Rightarrow c_1 = c_2 = c$. So, the equality $(\theta \circ \Phi)(P) = (\theta \circ \Phi_n)(P) = c$ for every θ implies that $\Phi_n(P) = \Phi(P)$. □

In the work [6] it was formulated and proved the Lemma 1 on extension of the linear functional $\varphi \in H_b(X)'$ to character $\psi \in M_b$. The following theorem is a generalization of the known lemma and is related to the study of extension of linear operator to the homomorphism.

Theorem 2. *Let $\Phi \in \mathcal{L}(H_b(A \otimes_\pi X), A)$ be a linear operator such that $\Phi(P) = 0$ for every $P \in \mathcal{P}(^m(A \otimes_\pi X), A) \cap A_{m-1}(A \otimes_\pi X)$, where m is a fixed positive integer and Φ_m be the nonzero restriction of Φ to $\mathcal{P}(^m(A \otimes_\pi X))$.*

Then there is a homomorphism $\Psi \in M_A(H_b(A \otimes_\pi X))$ such that its restrictions Ψ_k to $\mathcal{P}(^k(A \otimes_\pi X))$ satisfy the conditions: $\Psi_k = 0$ for all $k < m$ and $\Psi_m = \Phi_m$. Moreover, the radius functions of Ψ is calculated by the formula

$$\|\Phi_m\|^{1/m} \leq R(\Psi) \leq e\|\Phi_m\|^{1/m}.$$

Proof. For every polynomial $P \in \mathcal{P}(^{mk}(A \otimes_\pi X))$ we denote by $P_{(m)}$ the polynomial from $\mathcal{P}(^k \otimes^m_{s,\pi}(A \otimes_\pi X))$ such that $P_{(m)}(\overline{a}^{\otimes m}) = P(\overline{a})$.

Since $\Phi_m \neq 0$, there is an element $\omega \in (A \otimes_{s,\pi} X)'', \omega \neq 0$ such that for any m-homogeneous polynomial P,

$$\Phi(P) = \Phi_m(P) = \widetilde{P}_{(m)}(w), \qquad \|w\| = \|\Phi_m\|,$$

where $\widetilde{P}_{(m)}$ is the Aron-Berner extension of linear functional $P_{(m)}$ from $\otimes^m_{s,\pi}(A \otimes_\pi X)$ to $\otimes^m_{s,\pi}(A \otimes_\pi X)''$. For an arbitrary n-homogeneous polynomial $Q \in \mathcal{P}(^n(A \otimes_\pi X))$ we set

$$\Psi(Q) = \begin{cases} \widetilde{Q}_{(m)}(w) & \text{if } n = mk \text{ for some } k \geq 0, \\ 0 & \text{otherwise}, \end{cases} \tag{2}$$

where $\widetilde{Q}_{(m)}$ is the Aron-Berner extension of the k-homogeneous polynomial $Q_{(m)}$ from $\otimes^m_{s,\pi}(A \otimes_\pi X)$ to $\otimes^m_{s,\pi}(A \otimes_\pi X)''$.

Let (u_α) be a net from $\otimes^m_{s,\pi}(A \otimes_\pi X)$ which converges to w in the weak-star topology of $\otimes^m_{s,\pi}(A \otimes_\pi X)''$, where α belongs to an index set $\mathfrak{A}$. We can assume that every u_α has a representation $u_\alpha = \sum_{j \in \mathbb{N}}(a_{j,\alpha} \otimes_\pi x_{j,\alpha})^{\otimes m} = \sum_{j \in \mathbb{N}} p_{j,\alpha}^{\otimes m}$ for some $a_{j,\alpha} \in A, x_{j,\alpha} \in X$.

Now we will show that $\Psi(PQ) = \Psi(P)\Psi(Q)$ for any homogeneous polynomials P and Q.

1) Let us suppose first that $\deg(PQ) = mr + l$ for some integers $r \geq 0$ and $m > l > 0$. Then P or Q has degree equal to $mk + s$, $k \geq 0$, $m > s > 0$. Thus, by the definition $\Psi(PQ) = 0$ and $\Psi(P)\Psi(Q) = 0$.

2) Suppose now that for some integer $r \geq 0$ $\deg(PQ) = mr$. If $\deg P = mk$ and $\deg Q = mn$ for $k, n \geq 0$, then $\deg(PQ) = m(k+n)$ and

$$\Psi(PQ) = (\widetilde{PQ})_{(m)}(w) = \widetilde{P}_{(m)}(w)\widetilde{Q}_{(m)}(w) = \Psi(P)\Psi(Q).$$

3)Let at last $\deg P = mk + l$ and $\deg Q = mn + r$, $l, r > 0$, $l + r = m$. Write

$$\nu = \frac{1}{(\deg P + \deg Q)!} = \frac{1}{(m(k+n+1))!}.$$

Denote by F_{PQ} the symmetric multilinear map, associated with PQ. Then

$$\begin{aligned} &F_{PQ}\left(\overline{a}_1, \ldots, \overline{a}_{m(k+n+1)}\right) \\ &= \nu \sum_{\sigma \in \mathfrak{S}_{m(k+n+1)}} F_P\left(\overline{a}_{\sigma(1)}, \ldots, \overline{a}_{\sigma(mk+l)}\right) F_Q\left(\overline{a}_{\sigma(mk+l+1)}, \ldots, \overline{a}_{\sigma(m(k+n+1))}\right), \end{aligned}$$

where $\mathfrak{S}_{m(k+n+1)}$ is the group of permutations on $\{1, \ldots, m(k+n+1)\}$. Thus, for $\alpha_1, \ldots, \alpha_{k+n+1} \in \mathfrak{A}$ we have

$$\begin{aligned} \psi(PQ) = (\widetilde{PQ})_{(m)}(w) &= \lim_{\alpha_1, \ldots, \alpha_{k+n+1}} \widetilde{F}_{PQ_{(m)}}\left(u_{\alpha_1}, \ldots, u_{\alpha_{k+n+1}}\right) \\ &= \lim_{\alpha_1, \ldots, \alpha_{k+n+1}} \widetilde{F}_{PQ_{(m)}}\left(\sum_{j \in \mathbb{N}} p_{j,\alpha_1}^{\otimes m}, \ldots, \sum_{j \in \mathbb{N}} p_{j,\alpha_{k+n+1}}^{\otimes m}\right) \\ &= \nu \sum_{\sigma \in \mathfrak{S}_{m(k+n+1)}} \lim_{\alpha_{\sigma(1)}, \ldots, \alpha_{\sigma(k+n+1)}} \\ &\sum_{j_1, \ldots, j_{k+n+1} \in \mathbb{N}} F_P\left(\overline{a}^m_{j_{\sigma(1)}, \alpha_{\sigma(1)}}, \ldots, \overline{a}^m_{j_{\sigma(k)}, \alpha_{\sigma(k)}}, \overline{a}^l_{j_{\sigma(k+1)}, \alpha_{\sigma(k+1)}}\right) \\ &\times F_Q\left(\overline{a}^r_{j_{\sigma(k+1)}, \alpha_{\sigma(k+1)}}, \overline{a}^m_{j_{\sigma(k+2)}, \alpha_{\sigma(k+2)}}, \ldots, \overline{a}^m_{j_{\sigma(k+n+1)}, \alpha_{\sigma(k+n+1)}}\right). \end{aligned}$$

Fix some $\sigma \in \mathfrak{S}_{m(k+n+1)}$ and fix all $\overline{a}_{j_{\sigma(i)}, \alpha_{\sigma(i)}}$ for $i \leq k$ and for $i > k+1$. Then

$$\begin{aligned} \sum_{j_1, \ldots, j_{k+n+1} \in \mathbb{N}} \lim_{\alpha_{\sigma(k+1)}} F_P&\left(\overline{a}^m_{j_{\sigma(1)}, \alpha_{\sigma(1)}}, \ldots, \overline{a}^m_{j_{\sigma(k)}, \alpha_{\sigma(k)}}, \overline{a}^l_{j_{\sigma(k+1)}, \alpha_{\sigma(k+1)}}\right) \\ \times F_Q&\left(\overline{a}^r_{j_{\sigma(k+1)}, \alpha_{\sigma(k+1)}}, \overline{a}^m_{j_{\sigma(k+2)}, \alpha_{\sigma(n+2)}}, \ldots, \overline{a}^m_{j_{\sigma(k+n+1)}, \alpha_{\sigma(k+n+1)}}\right) = 0, \end{aligned}$$

because for a fixed $\overline{a}_{k_{\sigma(i)}, \alpha_{\sigma(i)}}$, $i \leq k$,

$$P_\sigma(y) := \sum_{j_1, \ldots, j_k, j_{k+2}, \ldots, j_{k+n+1} \in \mathbb{N}} F_P\left(\overline{a}^m_{j_{\sigma(1)}, \alpha_{\sigma(1)}}, \ldots, \overline{a}^m_{j_{\sigma(k)}, \alpha_{\sigma(k)}}, y^l\right)$$

is an l-homogeneous polynomial and for fixed $\overline{a}_{k_{\sigma(i)}, \alpha_{\sigma(i)}}$, $i > k+1$,

$$Q_\sigma(y) := \sum_{j_1, \ldots, j_k, j_{k+2}, \ldots, j_{k+n+1} \in \mathbb{N}} F_Q\left(y^r, \overline{a}^m_{j_{\sigma(k+2)}, \alpha_{\sigma(n+2)}}, \ldots, \overline{a}^m_{j_{\sigma(k+n+1)}, \alpha_{\sigma(k+n+1)}}\right)$$

is an r-homogeneous polynomial. Thus, $P_\sigma Q_\sigma \in \mathcal{A}_{m-1}(A \otimes_\pi X)$. Hence,

$$\lim_\alpha (P_\sigma Q_\sigma)_{(m)}(u_\alpha) = \Psi(P_\sigma Q_\sigma) = 0$$

for every fixed σ. Therefore, $\Psi(PQ) = 0$. On the other hand, $\Psi(P)\Psi(Q) = 0$ by the definition of Ψ. So, $\Psi(PQ) = \Psi(P)\Psi(Q)$.

Thus, we have defined the multiplicative operator Ψ on homogeneous polynomials. We can extend it by linearity and distributivity to a homomorphism on the algebra of all continuous polynomials $\mathcal{P}(A \otimes_\pi X)$.

If Ψ_n is the restriction of Ψ to $\mathcal{P}({}^n(A \otimes_\pi X))$, then $\|\Psi_n\| = \|w\|^{n/m}$ if n/m is a positive integer and $\|\Psi_n\| = 0$ otherwise. Hence, the series

$$\Psi = \sum_{n\in\mathbb{N}} \Psi_n$$

is a continuous homomorphism on $H_b(A \otimes_\pi X)$ by Theorem 1 and the radius function of Ψ can be computed by $R(\Psi) = \limsup\limits_{n\to\infty} \|\Psi_n\|^{1/n} \geq \limsup\limits_{n\to\infty} \|w\|^{n/mn} = \|w\|^{1/m} = \|\Phi_m\|^{1/m}$. On the other hand, $\|\Psi_n\| = \sup_{\|P\|=1} |\Psi_n(P)| = \sup_{\|P\|=1} |P_{(m)}(w)|$. Since

$$|P_{(m)}(w)| \leq \|w\|^{n/m}\|P_{(m)}\| \leq c(n, A \otimes_\pi X)\|w\|^{n/m}\|P\|,$$

we have

$$\|\Psi_n\| \leq c(n, A \otimes_\pi X)\|w\|^{n/m} \leq \frac{n^n}{n!}\|w\|^{n/m} = \frac{n^n}{n!}\|\Phi_m\|^{n/m}.$$

So $R(\Psi) \leq e\|\Phi_m\|^{1/m}$. The theorem is proved. □

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Досліджено гомоморфізми алгебри цілих функцій обмеженого типу на банахових просторах в комутативну банахову алгебру. Зокрема, запропоновано метод побудови гомоморфізмів, які є нулем на однорідних поліномах степеня, що не перевищує деяке фіксоване число n.

Ключові слова і фрази: Продовження Арона-Бернера, функціональне числення, алгебри аналітичних функцій в банахових просторах.