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MALYTSKA H.P., BURTNYAK I.V.

THE FUNDAMENTAL SOLUTION OF CAUCHY PROBLEM FOR A SINGLE EQUATION OF THE DIFFUSION EQUATION WITH INERTIA

In the paper it is found the explicit form of the fundamental solution of Cauchy problem for the equation of Kolmogorov type that has a finite number groups of spatial variables on which there is degeneration of parabolicity.

Key words and phrases: Kolmogorov equations, the fundamental solution, degenerate parabolic equations.

Vasyl Stefanyk Precarpathian National University, 57 Shevchenka str., 76018, Ivano-Frankivsk, Ukraine
E-mail: bvanya@meta.ua (Burtnyak I.V.)

INTRODUCTION

In this article we explore the fundamental solution of Cauchy problem (FSCP) or the diffusion equation with inertia, which depends on the inertia of many groups of spatial variables.

In 1934 based on a particular tangent process, Kolmogorov A.N. derived the diffusion equation with inertia [1]. Using the Fourier transform, Hormander L. found the fundamental solution of the equation [2].

To construction of fundamental solutions of the Cauchy problem for ultraparabolic equations where involved many authors, among them Weber M. [3], Il'in, A.M. [4], Oleinik O.A. [5], Eidelman S.D. [6], Ivasyshen S.D. [7] and their students. There were considered equations with one, two spatial groups of variables on which there is degeneration of parabolicity as well as equations having features on the time variable. Detailed analysis of the theory of degenerate parabolic equations in the appropriate time period is done in the works [8, 9]. We consider the equations which have the degeneration of parabolicity for arbitrary finite number of groups of spatial variables. This research is a continuation of works [10, 11].

1 NOTATIONS AND PROBLEM STATEMENT

Let $x := (x_{11}, x_{12}, \dots, x_{1n_1}; x_{21}, x_{22}, \dots, x_{2n_2}; \dots; x_{k1}, x_{k2}, \dots, x_{kn_k}; \dots; x_{p1}, x_{p2}, \dots, x_{pn_p}; x_{q1}, \dots, x_{m1})$, $q = p+1$, $n_1 \ge n_2 \ge \dots \ge n_p > 1$, $n_k \in \mathbb{N}$, $k = \overline{1,p}$, $p \in \mathbb{N}$, $m \ge p$, $\sum\limits_{k=1}^{p} n_k + m - p = n$, $x \in \mathbb{R}^n$, $\xi \in \mathbb{R}^n$. Consider the Cauchy problem

$$\partial_t u(t,x) - \sum_{k=1}^{p}\sum_{j=1}^{n_k} x_{kj}\partial_{x_{k\,j+1}} u(t,x) = \sum_{v=1}^{m} \partial^2_{x^2_{k1}} u(t,x), \tag{1}$$

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$$u(t,x)\,|_{t=\tau} = u_0(x), \quad 0 \le \tau < t \le T < +\infty, \quad x \in \mathbb{R}^n, \tag{2}$$

where $u_0(x)$ — sufficiently smooth finite function. Let us find fundamental solution of Cauchy problem (1), (2).

2 Construction of the fundamental solution of Cauchy problem (1), (2).

The solution of the problem (1), (2) we will seek in the form of inverse Fourier transform of unknown function $v(t,\xi)$, so

$$u(t,x) = (2\pi)^{-\frac{n}{2}} \int_{\mathbb{R}^n} \exp\{i(x,\xi)\}\, v(t,\xi)\, d\xi. \tag{3}$$

Since $\partial_t u(t,x) = F(\partial_t v(t,\xi))$, and

$$x_{kj}\partial_{x_{kj+1}} u(t,x) = F\left(-\xi_{kj+1}\partial_{\xi_{kj}} v(t,\xi)\right), \qquad \partial^2_{x^2_{k1}} u(t,x) = F\left(-\xi^2_{k1} v(t,\xi)\right),$$

the problem (1), (2) is reduced to the problem

$$\partial_t v(t,\xi) - \sum_{k=1}^{p}\sum_{j=1}^{n_k} \xi_{kj+1}\partial_{\xi_{kj}} v(t,\xi) = \sum_{k=1}^{m} \xi^2_{k1} v(t,\xi), \tag{4}$$

$$v(t,\xi)\,|_{t=\tau} = v_0(\xi), \quad 0 \le \tau < t \le T < +\infty, \quad \xi \in \mathbb{R}^n. \tag{5}$$

The following system corresponds to equation (4)

$$\begin{aligned} dt &= \frac{d\xi_{11}}{\xi_{12}} = \frac{d\xi_{12}}{\xi_{13}} = \dots = \frac{d\xi_{1n_1-1}}{\xi_{1n_1}} = \frac{d\xi_{21}}{\xi_{22}} = \dots = \frac{d\xi_{2n_2-1}}{\xi_{2n_2}} = \dots = \frac{d\xi_{k1}}{\xi_{k2}} = \dots \\ &= \frac{d\xi_{kn_k-1}}{\xi_{kn_k}} = \frac{d\xi_{p1}}{\xi_{p2}} = \dots = \frac{d\xi_{pn_p-1}}{\xi_{pn_p}} = \frac{dv}{-\sum\limits_{k=1}^{m} \xi^2_{k1} v}. \end{aligned} \tag{6}$$

Let us find $\sum\limits_{k=1}^{m} n_k + 1 - p$ independent integrals of the system (6). From $dt = \frac{d\xi_{kn_k-1}}{\xi_{kn_k}}$, $k = \overline{1,p}$, we obtain

$$\xi_{kn_k-1} = t\xi_{kn_k} + c_{kn_k-1}, \tag{7}$$

and from $dt = \frac{d\xi_{kn_k-2}}{\xi_{kn_k-1}}$, $k = \overline{1,p}$, using (7), we obtain

$$\xi_{kn_k-2} = \frac{t^2}{2!}\xi_{kn_k} + tc_{kn_k-1} + c_{kn_k-2}, \tag{8}$$

and so on. From $dt = \frac{d\xi_{kn_k-j}}{\xi_{kn_k-j+1}}$, we obtain

$$\xi_{kn_k-j} = \frac{t^j}{j!}\xi_{kn_k} + \frac{t^{j-1}}{(j-1)!}c_{kn_k-1} + \dots + c_{kn_k-2}. \tag{9}$$

Analogically, from $dt = \frac{d\xi_k}{\xi_{k2}}$, using $\xi_{k2} = \frac{t^{n_k-2}}{(n_k-2)!}\xi_{kn_k} + \frac{t^{n_k-3}}{(n_k-3)!}c_{kn_{k1}} + \dots + c_{k2}$, we obtain

$$\xi_{k1} = \frac{t^{n_k-1}}{(n_k-1)!}\xi_{kn_k} + \frac{t^{n_k-2}}{(n_k-2)!}c_{1n_k-1} + \dots + c_{k2} + c_{k1}. \tag{10}$$

Let us consider $dt = \frac{dv}{-\left[\sum\limits_{k=1}^{p}\left(\frac{t^{n_k-1}}{(n_k-1)!}\xi_{kn_k}+\frac{t^{n_k-2}}{(n_k-2)!}c_{1n_k-1}+\cdots+c_{k,1}\right)^2+\sum\limits_{k=p+1}^{m}\xi_{k1}^2\right]v}$, and find the integral

$$v = c\exp\left\{-\int\limits_{\tau}^{t}\sum_{k=1}^{p}\left(\frac{\beta^{n_k-1}}{(n_k-1)!}\xi_{kn_k}+\frac{\beta^{n_k-2}}{(n_k-2)!}c_{1n_k-1}+\cdots+c_{k1}\right)^2 d\beta-\sum_{k=p+1}^{m}\xi_{k1}^2\,(t-\tau)\right\}, \tag{11}$$

with $t > \tau$. The initial condition implies

$$\begin{gathered} v_0\left(\tfrac{\tau^{n_1-1}}{(n_1-1)!}\xi_{1\,n_1}+\tfrac{\tau^{n_1-2}}{(n_1-2)!}c_{1\,n_1-1}+\cdots+\tau c_{12}+c_{1\,1}\,;\ldots;\tau\xi_{1\,n_1}+c_{1\,n_1-1},\xi_{1\,n_1};\right.\\ \tfrac{\tau^{n_2-1}}{(n_2-1)!}\xi_{2\,n_2}+\tfrac{\tau^{n_2-2}}{(n_2-2)!}c_{2\,n_2-1}+\cdots+\tau c_{22}+c_{21};\ldots;\tau\xi_{2\,n_2}+c_{2\,n_2},\ldots\xi_{2\,n_2};\ldots;\tfrac{\tau^{n_p-1}}{(n_p-1)!}\xi_{p\;n_p}\\ \left.+\tfrac{\tau^{n_p-2}}{(n_p-2)!}c_{p\;n_p-1}+\ldots+\tau c_{p2}+c_{p\,1};\xi_{p+1\,1};\xi_{p+2\,1},\ldots,\xi_{m\,1}\right)=c, \end{gathered} \tag{12}$$

therefore

$$\begin{gathered} v=\exp\Big\{-\sum_{k=1}^{m}\xi_{k1}^2(t-\tau)-\sum_{k=1}^{p}\int\limits_{\tau}^{t}(\beta^{n_k-1}\xi_{kn_k}^2((n_k-1)!)^{-1}+\beta^{n_k-2}c_{kn_k-1}((n_k-2)!)^{-1}\\ +\cdots+\beta c_{k2}+c_{k1})^2d\beta\Big\}v_0\big(\xi_{1\,n_1}\tau^{n_1-1}\left((n_1-1)!\right)^{-1}+\sum_{l=1}^{n_1-1}\left((n_1-1-l)!\right)^{-1}c_{1\,n_1-l}\\ \tau^{n_1-1-l},\ldots,\tau\xi_{1\,n_1}+c_{1\,n_1-1};\xi_{1\,n_1};\ldots;\xi_{p\,n_p}\tau^{n_p-1}\left((n_p-1)!\right)^{-1}+\sum_{l=1}^{n_p-1}\left((n_p-1-l)!\right)^{-1}\\ c_{pn_p-l}\tau^{n_p-1-l},\ldots,\tau\xi_{p\,n_p}+c_{p\,n_p-1};\xi_{p\,n_p};\xi_{p\,+11},\ldots,\xi_{m1}\big) \end{gathered} \tag{13}$$

Replace $c_{1n_1-1},\ldots,c_{1\,1};c_{2n_2-1},\ldots,c_{2\,1};\ldots;c_{pn_p-1},\ldots,c_{p\,1}$ with their values that are found from the system of first integrals, $k=\overline{1,p}$;

$$\begin{cases} \xi_{k\,n_k-1}=t\xi_{k\,n_k}+c_{k\,n_k-1},\\ \xi_{k\,n_k-2}=\frac{t^2}{2!}\xi_{k\,n_k}+tc_{k\,n_k-1}+c_{k\,n_k-2},\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \xi_{k\,n_k-j}=\frac{t^j}{j!}\xi_{k\,n_k}+\frac{t^{j-1}}{(j-1)!}c_{k\,n_k-1}+\cdots+tc_{k\,n_k-j+1}+c_{k\,n_k-j},\\ \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\\ \xi_{k\,1}=\frac{t^{n_k-1}}{(n_k-1)!}\xi_{k\,n_k}+\frac{t^{n_k-1}}{(n_k-1)!}c_{k\,n_k-1}+\cdots+tc_{k\,2}+c_{k\,1}. \end{cases}$$

Therefore $c_{k\,n_k-1}=\xi_{k\,n_k-1}-t\xi_{k\,n_k}$, $c_{k\,n_k-2}=\xi_{k\,n_k-2}-t\xi_{k\,n_k-1}+\frac{t^2}{2!}\xi_{k\,n_k},\ldots$ Let

$$c_{k\,n_k-j}=\xi_{k\,n_k-j}-t\xi_{k\,n_k-j+1}+\cdots+\frac{(-t)^j}{j!}\xi_{k\,n_k}. \tag{14}$$

Let us show that

$$c_{k\,n_k-(j+1)}=\xi_{k\,n_k-(j+1)}-t\xi_{k\,n_k-j}+\cdots+\frac{(-t)^{j+1}}{(j+1)!}\,\xi_{k\,n_k}. \tag{15}$$

Indeed, since $c_{k\,n_k-(j+1)}=\xi_{k\,n_k-(j+1)}-\frac{t^{j+1}}{(j+1)!}\,\xi_{k\,n_k}-\frac{t^j}{j!}\,c_{k\,n_k-1}-\cdots-\frac{t^{j-l+1}}{(j-l+1)!}\,c_{k\,n_k-l}-tc_{k\,n_k-j}$, using (14), we have

$$\begin{gathered} c_{k\,n_k-(j+1)}=\xi_{k\,n_k-(j+1)}-\tfrac{t^{j+1}}{(j+1)!}\,\xi_{k\,n_k}-\tfrac{t^j}{j!}\,\left(\xi_{k\,n_k-1}-t\xi_{k\,n_k}\right)-\tfrac{t^{j-l+1}}{(j-l+1)!}\,\Big(\xi_{k\,n_k-l}-t\xi_{k\,n_k-l+1}\\ +\tfrac{t^2}{2!}\xi_{k\,n_k-l+2}+\tfrac{(-t)^\mu}{\mu!}\xi_{k\,n_k-l+\mu}+\cdots+\tfrac{(-t)^l}{l!}\xi_{k\,n_k}\Big)-\cdots-t\left(\xi_{k\,n_k-j}-t\xi_{k\,n_k-j+1}+\tfrac{t^2}{2!}\right. \end{gathered}$$

$\xi_{k\,n_k-j+2}+\dots+\frac{(-t)^j}{j!}\xi_{k\,n_k})=\xi_{k\,n_k-(j+1)}-t\,\xi_{k\,n_k-j}+\frac{t^2}{2!}\xi_{k\,n_k-l+2}+\dots+\frac{(-t)^{j-l+1}}{(j-l+1)!}\xi_{k\,n_k-l}$
$\left[(-1)\,(-1)^{j-l+1}-C^1_{j-l+1}\,(-1)^{j-l}-C^2_{j-l+1}\,(-1)^{j-l-1}-\dots-C^{j-l}_{j-l+1}\,(-1)\right]+\xi_{k\,n_k-1}\frac{t^j}{j!}$
$\left[-(-1+1)^j+1\right]+\xi_{k\,n_k}\frac{(-t)^{j+1}}{(j+1)!}\left[-(-1+1)^j+1\right]$, where $C^l_j=\frac{j!}{(j-l)!l!}$, $j\in\mathbb{N}\bigcup\{0\}$.

We have established (15), hence (14) is correct. Using (14), the formula (13) is reduced to the form

$$\begin{gathered}
v\,(t,\xi)=\exp\Bigg\{-\sum_{k=1}^{m}\xi^2_{k1}\,(t-\tau)-\int_{\tau}^{t}\sum_{k=1}^{p}\Big(\xi_{k1}+(\beta-t)\,\xi_{k2}+\dots+(\beta-t)^{n_k-1}\\
((n_k-1)!)^{-1}\,\xi_{kn_k}\Big)^2 d\beta\Bigg\}\,v_0\,(\xi_{1\,1}+(\tau-t)\,\xi_{12}+\dots+\frac{(\tau-t)^{n_1-1}}{(n_1-1)!}\xi_{1\,n_1},\xi_{1\,2}+(\tau-t)\\
\xi_{1\,3}+\dots+\frac{(\tau-t)^{n_1-2}}{(n_1-2)!}\xi_{1\,n_1},\dots,\xi_{1\,n_1-1}+(\tau-t)\,\xi_{1\,n_1},\xi_{1\,n_1};\xi_{21}+(\tau-t)\,\xi_{22}+\dots\\
+\frac{(\tau-t)^{n_2-1}}{(n_2-1)!}\xi_{2\,n_2},\dots,\xi_{2\,n_2-1}+(\tau-t)\,\xi_{2\,n_2},(\tau-t)\,,\dots,\xi_{p\,1}+(\tau-t)\,\xi_{p2}+\dots\\
+\frac{(\tau-t)^{n_p-1}}{(n_p-1)!}\xi_{p\,n_p},\dots,\xi_{p\,n_p-1}+(\tau-t)\,\xi_{p\,n_p},\xi_{p\,n_p};\xi_{p+1\,1},\dots,\xi_{m\,1}\Big)\,.
\end{gathered}\tag{16}$$

From (16) we find

$$\begin{gathered}
u\,(t,x)=\frac{1}{(2\pi)^{\frac{n}{2}}}\int_{\mathbb{R}^n}\exp\Bigg\{i\,(x,\xi)-\sum_{k=1}^{m}\xi^2_{k1}\,(t-\tau)-\sum_{k=1}^{p}\int_{\tau}^{t}\Big((\beta-t)^{n_k-1}\,((n_k-1)!)^{-1}\\
\xi_{kn_k}+(\beta-t)^{n_k-2}\,\xi_{kn_k-1}\,((n_k-2)!)^{-1}+\dots+(\beta-t)\,\xi_{k\,2k}+\xi_{k\,1}\Big)^2 d\beta\Bigg\}\,v_0\,(\xi_{11}\\
+(\tau-t)\,\xi_{12}+\dots+\xi_{1\,n_1}\,(\tau-t)^{n_1-1}\,((n_1-1)!)^{-1},\dots\xi_{1\,n_1-1}+(\tau-t)\,\xi_{1\,n_1},\xi_{1\,n_1},\dots,\\
\xi_{p\,1}+(\tau-t)\,\xi_{p2}+\dots+(\tau-t)^{n_p-1}\,\left((n_p-1)!\right)^{-1},\dots,\xi_{pn_p-1}+(\tau-t)\,\xi_{pn_p},\xi_{pn_p},\\
\xi_{p+1\,1},\dots,\xi_{m\,1})\,d\xi
\end{gathered}\tag{17}$$

Let us make the change of variables in the integral (17)

$$\begin{cases}
\xi_{k1}+(\tau-t)\,\xi_{k\,2}+\dots+(\tau-t)^{n_k-1}\,\xi_{kn_k}\,((n_k-1)!)^{-1}=\alpha_{k1},\\
\dots\dots\dots\dots\dots\\
\xi_{k\,j}+(\tau-t)\,\xi_{k\,j+1}+\dots+(\tau-t)^{n_k-j}\,\xi_{kn_k}\,((n_k-1)!)^{-1}=\alpha_{kj},\\
\dots\dots\dots\dots\dots\\
\xi_{k\;n_k-1}+(\tau-t)\,\xi_{k\,n_k}=\alpha_{k\;n_k-1},\\
\xi_{k\;n_k}=\alpha_{k\,n_k},\\
\xi_{p+1\;1}=\alpha_{p+1\;\;1},\\
\dots\dots\dots\dots\dots\\
\xi_{m1}=\alpha_{m1},\qquad k=\overline{1,p}.
\end{cases}\tag{18}$$

From (18) we have

$$\begin{cases}
\xi_{k\,n_k-1}=\alpha_{k\,n_k-1}-(\tau-t)\,\alpha_{k\,n_k},\\
\xi_{k\,n_k-2}=\alpha_{k\,n_k-2}-(\tau-t)\,\alpha_{k\,n_k-1}+\frac{(t-\tau)^2}{2!}\alpha_{k\,n_k},\\
\dots\dots\dots\dots\dots\\
\xi_{k\,n_k-j}=\alpha_{k\,n_k-j}-(t-\tau)\,\alpha_{k\,n_k-j+1}+\dots+\frac{(t-\tau)^j}{j!}\alpha_{k\,n_k},\\
\dots\dots\dots\dots\dots\\
\xi_{k1}=\alpha_{k1}-(t-\tau)\,\alpha_{k2}+\dots+\frac{(t-\tau)^{n_1-1}}{(n_1-1)!}\alpha_{kn_k},\quad k=\overline{1,p}.
\end{cases}\tag{19}$$

Therefore

$$u(t,x) = (2\pi)^{-\frac{n}{2}} \int\limits_{\mathbb{R}^n} \exp\left\{ \sum_{k=1}^{p} \sum_{j=1}^{n_k-1} i x_{k_1 j} \left(\alpha_{k_1 j} + (t-\tau)\alpha_{k_2 j+1} + \cdots + \frac{(t-\tau)^{n_k-j}}{(n_k-j)!} \xi_{k n_k} \right) \right.$$
$$+ \sum_{k=p+1}^{m} i x_{k1}\alpha_{k1} + \sum_{k=1}^{p} i x_{k n_k} \xi_{k n_k} - \sum_{k=p+1}^{m} \alpha_{k1}^2 (t-\tau) - \sum_{k=1}^{p} \int\limits_{\tau}^{t} \Big((\beta-\tau)^{n_k-1} \left((n_k-1)!\right)^{-1} \tag{20}$$
$$\left. \alpha_{k n_k} + (\beta-\tau)^{n_k-2} \alpha_{k n_{k-}-1} \left((n_k-2)!\right)^{-1} + \cdots + \alpha_{k,1} \Big)^2 d\beta \right\} v_0(\alpha) d\alpha.$$

Since $v_0(\alpha) = (2\pi)^{-\frac{n}{2}} \int\limits_{R^n} \exp\{-i(y,\alpha)\} u_0(y) dy$, then $u(t,x) = \int\limits_{R^n} G(t-\tau, x, \xi) u_0(\xi) d\xi$, where $G(t-\tau, x, \xi)$ is FSCP. Hence, from (20) we obtain the formula $G(t-\tau, x, \xi)$.

3 The fundamental solution of Cauchy problem

For $t > \tau$, $x \in \mathbb{R}^n$, $\xi \in \mathbb{R}^n$ from the formula (20) we have

$$G(t-\tau,\, x,\, \xi) = (2\pi)^{-n} \int\limits_{\mathbb{R}^n} \exp\left\{ - \sum_{k=1}^{p} i \left[(x_{k1} - \xi_{k1})\, \alpha_{k1} + (x_{k2} - \xi_{k2} + (t-\tau)\, x_{k1}) \right.\right.$$
$$\alpha_{k2} + \cdots + \left(x_{kj} - \xi_{kj} + x_{kj-1} (t-\tau) + \cdots + \tfrac{(t-\tau)^{j-1}}{(j-1)!} x_{k1} \right) \alpha_{kj} + \cdots + \big(x_{k\, n_k-1}$$
$$- \xi_{k\, n_k-1} + x_{k\, n_k-2} (t-\tau) + \cdots + \tfrac{(t-\tau)^{n_k-2}}{(n_k-2)!} x_{n1} \Big) \alpha_{k\, n_k-1} + \big(x_{k\, n_k} - \xi_{k\, n_k} + (t-\tau) \tag{21}$$
$$\left. x_{k\, n_k-1} + \cdots + \tfrac{(t-\tau)^{n_k-1}}{(n_k-1)!} x_{n1} \Big) \alpha_{k\, n_k} \right] + \sum_{k=p+1}^{m} i\, (x_{k1} - \xi_{k1})\, \alpha_{k1}$$
$$\left. - \sum_{k=p+1}^{m} \alpha_{k1}^2 \,(t-\tau) - \sum_{k=1}^{p} \int\limits_{\tau}^{t} \left(\alpha_{k1} + (\beta-\tau)\, \alpha_{k2} + \cdots + \frac{(\beta-\tau)^{n_k-1}}{(n_k-1)\,!} \alpha_{k\, n_k} \right)^2 d\beta \right\} d\alpha.$$

In order to find (21), consider the integral

$$I_k(t-\tau, \alpha) := \int_{\tau}^{t} \left(\alpha_{k1} + (\beta-\tau)\, \alpha_{k2} + \cdots + \frac{(\beta-\tau)^{n_k-1}}{(n_k-1)\,!} \alpha_{k\, n_k} \right)^2 d\beta. \tag{22}$$

Making the change of variables $\theta = \frac{\beta-\tau}{t-\tau}$, we have

$$I_k(t-\tau, \alpha) = (t-\tau) \int_{0}^{1} \left(\alpha_{k1} + \theta\, (t-\tau)\, \alpha_{k2} + \cdots + \frac{(\theta\, (t-\tau))^{n_k-1}}{(n_k-1)\,!} \alpha_{k\, n_k} \right)^2 d\theta.$$

In (21) let us replace $\alpha'_{k1}\,(t-\tau)^{-\frac{1}{2}} = \alpha_{k1}, \ldots, \alpha'_{k1}\,(t-\tau)^{-\frac{2n_k-1}{2}}\,((n_k-1)!) = \alpha_{kn_k}$, $k = \overline{1,p}$, $\alpha'_{k1}(t-\tau)^{-\frac{1}{2}} = \alpha_{k1}$, $k = \overline{p+1,m}$, and $\alpha' = \alpha$, then we obtain

$$
\begin{aligned}
G\left(t-\tau;x;\xi\right) = (2\pi)^{-n}\int_{\mathbb{R}^n} \exp\Bigg\{ -\sum_{k=1}^{p} i\Big[\left(x_{k1}-\xi_{k1}\right)\,\alpha_{k1}\left(t-\tau\right)^{-\frac{1}{2}} + \left(x_{k2}-\xi_{k2}\right. \\
\left. + \left(t-\tau\right)x_{k1}\right)\alpha_{k2}\left(t-\tau\right)^{-\frac{3}{2}} + \dots + \left(x_{kj}-\xi_{kj}+\left(t-\tau\right)x_{kj-1}+\dots+\frac{\left(t-\tau\right)^{j-1}}{(j-1)!}x_{k1}\right) \\
\alpha_{kj}\left(t-\tau\right)^{-\frac{2j-1}{2}}(j-1)! + \dots + \left(x_{kn_k}-\xi_{kn_k}+\left(t-\tau\right)x_{kn_k-1}+\dots+\frac{\left(t-\tau\right)^{n_k-1}}{(n_k-1)!}x_{k1}\right) \\
\alpha_{kn_k}\left(t-\tau\right)^{-\frac{2n_k-1}{2}}\left(n_k-1\right)!\Big] - \sum_{k=p+1}^{m}\left(x_{k1}-\xi_{k1}\right)\alpha_{k1}\left(t-\tau\right)^{-\frac{1}{2}} - \sum_{k=p+1}^{m}\alpha_{k1}^2 \\
-\sum_{k=1}^{p}\int_0^1\left(\alpha_{k1}+\theta\alpha_{k2}+\dots+\theta^{n_k-1}\alpha_{kn_k}\right)^2 d\theta\Bigg\}\, d\alpha\left(t-\tau\right)^{-\mu}\prod_{k=1}^{p}\prod_{k=0}^{n_k-1}k!,
\end{aligned}
\tag{23}
$$

where $\mu = \frac{m}{2} + \frac{\sum_{k=1}^{p}\left(n_k-1\right)^2}{2}$.

Let us calculate the integral $I = \int_0^1 \left(\alpha_{k1}+\theta\alpha_{k2}+\dots+\theta^{n_k-1}\alpha_{kn_k}\right)^2 d\theta$:

$$
\begin{aligned}
I = \alpha_{k1}^2 + \frac{\alpha_{k2}^2}{3} + \frac{\alpha_{k3}^2}{5} + \dots + \frac{\alpha_{kn_k}^2}{2n_k-1} + \alpha_{k1}\alpha_{k2} + \dots + 2\frac{\alpha_{k1}\alpha_{kj}}{j} + \dots + 2\frac{\alpha_{k1}\alpha_{kn_k}}{n_k} + \dots \\
+2\frac{\alpha_{kj}\alpha_{k(j+\nu)}}{2j+\nu-1} + \dots + \frac{\alpha_{kn_k-1}\alpha_{kn_k}}{n_k-1}.
\end{aligned}
\tag{24}
$$

After making perfect squares in (24) we obtain

$$
\begin{aligned}
I = \left(\sum_{j=1}^{n_k}\frac{\alpha_{kj}}{j}\right)^2 + 3\left(\sum_{j=2}^{n_k}\frac{\alpha_{kj}\left(j-1\right)}{j\left(j+1\right)}\right)^2 + 5\left(\sum_{j=3}^{n_k}\frac{\alpha_{kj}\left(j-1\right)\left(j-2\right)}{j\left(j+1\right)\left(j+2\right)}\right)^2 + \dots + \left(2k_0-1\right) \\
\left(\sum_{j=k_0}^{n_k-1}\frac{\alpha_{kj}\left(j-1\right)\dots\left(j-k_0+1\right)}{j\left(j+1\right)\dots\left(j+k_0-1\right)}\right)^2 + \dots + \left(2n_k-1\right)\left(\frac{\alpha_{kn_k}\left(n_k-1\right)!}{n_k\dots\left(2n_k-1\right)}\right)^2.
\end{aligned}
\tag{25}
$$

Using (25), let us make the change of variables in the integral (23)

$$
\begin{cases}
\sum\limits_{j=1}^{n_k}\frac{\alpha_{kj}}{j} = s_{k1}, \\
\sum\limits_{j=2}^{n_k}\frac{\alpha_{kj}(j-1)}{j(j+1)} = s_{k2}, \\
\dots\dots\dots\dots\dots \\
\sum\limits_{j=k_0}^{n_k-1}\frac{\alpha_{kj}(j-1)\dots(j-k_0+1)}{j(j+1)\dots(j+k_0-1)} = s_{kj}, \\
\dots\dots\dots\dots\dots \\
\frac{\alpha_{kn_k}(n_k-1)!}{n_k\dots(2n_k-1)} = s_{kn_k}, \qquad k=\overline{1,p}, \\
\alpha_{k1} = s_{k1}, \qquad k=\overline{p+1,m}.
\end{cases}
\tag{26}
$$

Solving the equation (26) with respect to α, we obtain

$$
\begin{aligned}
&\alpha_{k1} = s_{k1} - 3s_{k2} + 5s_{k3} - 7s_{k4} + \dots + (-1)^{n_k-1}(2n_k-1)\,s_{kn_k},\\
&\tfrac{\alpha_{k1}}{2\cdot 3} = s_{k2} - 5s_{k3} + \tfrac{4\cdot 7}{2!}s_{k4} - \tfrac{4\cdot 5\cdot 9}{3!}s_{k5} + \tfrac{4\cdot 5\cdot 6\cdot 11}{4!}s_{k6} + \dots + \tfrac{(-1)^{n_k-1}}{(n_k-2)}4\cdot 5\cdot \dots \cdot n_k\,(2n_k-1)\,s_{kn_k},\\
&\dots\dots\dots\dots\dots\\
&\tfrac{\alpha_{kj}(j-1)!}{j(j+1)\dots(2j-1)} = s_{kj} - (2j+1)\,s_{kj+1} + \tfrac{2j(2j+3)}{2!}s_{kj+2} - \tfrac{2j(2j+1)(2j+5)}{3!}s_{kj+3}\\
&+\tfrac{2j(2j+1)(2j+2)(2j+7)}{4!}s_{kj+4} + \dots + \tfrac{(-1)^{\nu-j}2j(2j+1)\dots(j+\nu-2)(2\nu-1)}{(\nu-j)!}s_{k\nu} + \dots\\
&+\tfrac{(-1)^{n_k-j}2j(2j+1)\dots(n_k+j-2)(2n_k-1)}{(n_k-j)!}s_{kn_k}, \dots,\\
&\tfrac{\alpha_{kn_k-1}(n_k-2)!}{(n_k-1)\dots(2n_k-3)} = s_{kn_k-1} - (2n_k-1)\,s_{kn_k}, \qquad \tfrac{\alpha_{kn_k-1}(n_k-1)!}{n_k(n_k+1)(2n_k-1)} = s_{kn_k}, \qquad k=\overline{1,p},\\
&s_{k1} = \alpha_{k1}, \qquad\qquad k=\overline{p+1,m}.
\end{aligned}
$$

From this system we find $\alpha_{kj}\,(j-1)!$, $j=\overline{1,p}$, and substitute it in (23). Therefore $G\,(t-\tau;x;\xi)$ has the form

$$
\begin{aligned}
&G\,(t-\tau;x;\xi) = (2\pi)^{-n}\int_{\mathbb{R}^n}\exp\Bigg\{-\sum_{\nu=1}^{p}\Bigg[\sum_{k=1}^{n_\nu}(2k-1)\,s_{\nu k}^2 - \Big(\sum_{k=1}^{n_\nu}(-1)^{k-1}(2k-1)\,s_{\nu k}\Big)\\
&i\,(x_{\nu 1}-\xi_{\nu 1})\,(t-\tau)^{-\frac{1}{2}} - 3!i\,(t-\tau)^{-\frac{3}{2}}\,(x_{\nu 2}-\xi_{\nu 2}+(t-\tau)\,x_{\nu 1})\,\Big(s_{\nu 2}-5s_{\nu 3}+\tfrac{4\cdot 7}{2!}s_{\nu 4}\\
&-\tfrac{4\cdot 5\cdot 9}{3!}s_{\nu 5} - \dots - \tfrac{(-1)^{n_\nu-2}}{(n_\nu-2)!}4\cdot 5\dots n_\nu\,(2n_\nu-1)\,s_{\nu n_\nu}\Big) - \dots - k\dots(2k-1)\,(t-\tau)^{-\frac{2k-1}{2}}\,i\\
&\Big(s_{\nu k} - (2k+1)\,s_{\nu k+1} + \tfrac{2k(2k+3)}{2!}s_{\nu k+2} - \tfrac{2k(2k+1)(2k+5)}{3!}s_{\nu k+3} + \dots + \tfrac{2k(2k+1)(2k+2)(2k+7)}{4!}\\
&s_{\nu k+4} + \dots + (-1)^{j-k}\,s_{\nu j}\tfrac{2k(2k+1)\dots(j+k-2)(2j-1)}{(j-k)!} + \dots + \tfrac{(-1)^{n_\nu-k}2k(2k+1)\dots(n_\nu+k-2)(2n_\nu-1)}{(n_\nu-k)!}\\
&s_{\nu n_\nu}\Big(\sum_{j=0}^{k-1}x_{\nu k-j}\tfrac{(t-\tau)^j}{j!} - \xi_{\nu k}\Big)\Big) + \dots + n_\nu\,(n_\nu-1)\dots(2n_\nu-1)\,(t-\tau)^{-\frac{(2n_\nu-1)}{2}}\,is_{\nu n_\nu}\\
&-\Big(\sum_{j=0}^{n_\nu-1}x_{n_\nu-j}\tfrac{(t-\tau)^j}{j!} - \xi_{\nu n_\nu}\Big)\sum_{k=p+1}^{m}s_{k1}^2 + i\sum_{k=p+1}^{m}(x_{k1}-\xi_{k1})\,s_{k1}\,(t-\tau)^{-\frac{1}{2}}\Bigg]\Bigg\}ds\\
&(t-\tau)^{-\mu}\prod_{\nu=1}^{p}\prod_{j=1}^{n_\nu}k\,(k+1)\dots(2k-1)\,.
\end{aligned}
\tag{27}
$$

In (27) we group the similar terms with respect to $s_{\nu\,j}$, we obtain

$$
\begin{aligned}
&G(t-\tau,\,x,\,\xi) = (2\pi)^{-n}\int_{\mathbb{R}^n}\exp\Bigg\{-i\sum_{\nu=1}^{p}\Big[s_{\nu 1}(t-\tau)^{-\frac{1}{2}}(x_{\nu 1}-\xi_{\nu 1}) + s_{\nu 2}(t-\tau)^{-\frac{3}{2}}3!\\
&(x_{\nu 2}-\xi_{\nu 2}+(x_{\nu 1}+\xi_{\nu 1})(t-\tau)2^{-1}) + s_{\nu 3}(t-\tau)^{-\frac{5}{2}}3\cdot 4\cdot 5(x_{\nu 3}-\xi_{\nu 3}+(x_{\nu 2}+\xi_{\nu 2})\\
&(t-\tau)2^{-1}+(x_{\nu 1}-\xi_{\nu 1})(t-\tau)^2 12^{-1}) + s_{\nu 4}(t-\tau)^{-\frac{7}{2}}4\cdot 5\cdot 6\cdot 7(x_{\nu 4}-\xi_{\nu 4}\\
&+(x_{\nu 3}+\xi_{\nu 3})(t-\tau)2^{-1}+(x_{\nu 2}-\xi_{\nu 2})(t-\tau)^2 10^{-1}+(t-\tau)^2(x_{\nu 1}+\xi_{\nu 1})\;120^{-1}) + \dots\\
&+n_\nu(n_\nu+1)\dots(2n_\nu-1)s_{\nu n_\nu}\Big(\sum_{j=0}^{n_\nu-1}x_{\nu n_v-j}\tfrac{(t-\tau)^j}{j!} - \xi_{\nu n_v} - 2^{-1}(t-\tau)\\
&\Big(\sum_{j=0}^{n_\nu-2}\tfrac{x_{\nu n_v-1-j}(t-\tau)^j}{j!} - \xi_{\nu n_v-1}\Big) + (t-\tau)^2\Big(\sum_{j=0}^{n_\nu-3}\tfrac{x_{n_v-2-j}(t-\tau)^j}{j!} - \xi_{\nu n_v-2}\Big)\tfrac{(n_\nu-2)}{4(2n_\nu-3)} + \dots\\
&+(-1)^{n_v-k}\left((n_\nu-k)\,!\right)^{-1}(t-\tau)^{n_v-k}2k\,(2k+1)\dots(2k+(n_\nu-k)-2)\\
&(2k+2(n_\nu-k)-1)(n_\nu\dots(2n_\nu-1))^{-1}\Big(\sum_{j=0}^{k-1}\tfrac{x_{\nu k-1-j}(t-\tau)^j}{j!} - \xi_{\nu k}\Big) + \dots + (-1)^{n_v-2}\\
&(t-\tau)^{n_v-2}(x_{\nu 2}-\xi_{\nu 2}+(t-\tau)x_{\nu 1})\left(2\,(n_\nu+1)\dots(2n_\nu-3)\,!\right)^{-1} + (-1)^{n_v-1}\\
&(t-\tau)^{n_v-1}(x_{\nu 1}-\xi_{\nu 1})(n_\nu\dots(2n_\nu-2)\,!)^{-1}\Big)\Big] - \sum_{\nu=1}^{p}\sum_{k=1}^{n_\nu}(2k-1)\,s_{\nu k}^2
\end{aligned}
\tag{28}
$$

$$\sum_{\nu=p+1}^{m}\left[s_{\nu 1}^{2}-i\left(x_{\nu 1}-\xi_{\nu 1}\right)s_{\nu 1}\left(t-\tau\right)^{-\frac{1}{2}}\right]\Big\}ds\left(t-\tau\right)^{-\mu}\prod_{\nu=1}^{p}\prod_{k=1}^{n_\nu}k\left(k+1\right)\dots\left(2k-1\right).$$

Analyzing (28), we obtain that $G(t-\tau,\,x,\,\xi)$ for $t>\tau$, $x\in\mathbb{R}^n$, $\xi\in\mathbb{R}^n$ is the Fourier transform of the function s

$$(2\pi)^{-\frac{n}{2}}\exp\Big\{-\sum_{\nu=1}^{p}\sum_{k=1}^{n_p}\left(2k-1\right)s_{\nu k}^{2}-\sum_{\nu=p+1}^{m}s_{\nu 1}^{2}\Big\}\left(t-\tau\right)^{-\mu}\prod_{\nu=1}^{p}\prod_{k=1}^{n_\nu}k\left(k+1\right)\dots\left(2k-1\right),$$

thus

$$\begin{aligned}
&G(t-\tau,\,x,\,\xi)=(2\sqrt{\pi})^{-n}\exp\Big\{-\sum_{\nu=1}^{m}|x_{\nu 1}-\xi_{\nu 1}|^{2}4^{-1}(t-\tau)^{-1}-\sum_{\nu=1}^{p}3\Big[\Big|x_{\nu 2}-\xi_{\nu 2}\\
&+\left(x_{\nu 1}+\xi_{\nu 1}\right)\left(t-\tau\right)2^{-1}\Big|^{2}\left(t-\tau\right)^{-3}+180\big|x_{\nu 3}-\xi_{\nu 3}+\left(x_{\nu 2}+\xi_{\nu 2}\right)\left(t-\tau\right)2^{-1}\\
&+\left(x_{\nu 1}-\xi_{\nu 1}\right)\left(t-\tau\right)^{2}12^{-1}\big|^{2}\left(t-\tau\right)^{-5}+25200\big|x_{\nu 4}-\xi_{\nu 4}+\left(x_{\nu 3}+\xi_{\nu 3}\right)\left(t-\tau\right)2^{-1}\\
&+\left(x_{\nu 2}-\xi_{\nu 2}\right)\left(t-\tau\right)^{2}10^{-1}+\left(x_{\nu 1}-\xi_{\nu 1}\right)\left(t-\tau\right)^{3}120^{-1}\big|^{2}\left(t-\tau\right)^{-7}+\dots+\left(k-1\right)^{2}\\
&k^{2}\dots(2k-3)^{2}(2k-1)(t-\tau)^{-(2k-1)}\Big|\sum_{j=0}^{k-1}\tfrac{x_{\nu k-j}(t-\tau)^{j}}{j!}-\xi_{\nu k}-\Big(\sum_{j=0}^{k-2}\tfrac{x_{\nu k-1-j}(t-\tau)^{j}}{j!}\\
&-\xi_{\nu k-1}\Big)\left(t-\tau\right)2^{-1}+\dots+\left(-1\right)^{k-l}\tfrac{(t-\tau)^{(k-l)}}{(k-l)!}\tfrac{2l(2l+1)\dots(2l+(k-l)-2)(2l+2(k-l)-1)}{k\dots(2k-1)}\\
&\Big(\sum_{j=0}^{l-1}\tfrac{x_{\nu l-j}(t-\tau)^{j}}{j!}-\xi_{\nu l}\Big)+\dots+\tfrac{(-1)^{k-2}(t-\tau)^{(k-2)}(x_{\nu 2}-\xi_{\nu 2}+(t-\tau)x_{\nu 1})}{2(k+1)\dots(2k-3)}+\tfrac{(-1)^{k-1}(t-\tau)^{(k-1)}}{k\dots(2k-2)}\\
&(x_{\nu 1}-\xi_{\nu 1})\Big|^{2}+\dots+\left(n_\nu-1\right)^{2}n_\nu^{2}\left(n_\nu+1\right)^{2}\dots\left(2n_\nu-3\right)^{2}\left(2n_\nu-1\right)(t-\tau)^{-(2n_\nu-1)}\\
&\Big|\sum_{j=0}^{n_\nu-1}\tfrac{x_{\nu n_\nu-j}(t-\tau)^{j}}{j!}-\xi_{\nu n_\nu}-\left(t-\tau\right)2^{-1}\Big(\sum_{j=0}^{n_\nu-2}\tfrac{x_{\nu n_\nu-1-j}(t-\tau)^{j}}{j!}-\xi_{\nu n_\nu-1}\Big)+\left(t-\tau\right)^{2}\\
&\Big(\sum_{j=0}^{n_\nu-3}\tfrac{x_{\nu n_\nu-2-j}(t-\tau)^{j}}{j!}-\xi_{\nu n_\nu-2}\Big)\tfrac{(n_\nu-2)}{4(2n_\nu-3)}+\dots+(-1)^{n_\nu-k}\Big(\sum_{j=0}^{k-1}\tfrac{x_{\nu k-j}(t-\tau)^{j}}{j!}-\xi_{\nu k}\Big)\\
&\tfrac{(t-\tau)^{n_\nu-k}}{(n_\nu-k)!}\tfrac{2k\dots(n_\nu+k-2)}{n_\nu\dots(2n_\nu-2)}\Big(\sum_{j=0}^{k-1}\tfrac{x_{\nu k-j}(t-\tau)^{j}}{j!}-\xi_{\nu k}\Big)+\dots+\tfrac{(-1)^{n_\nu-2}(t-\tau)^{n_\nu-2}(x_{\nu 2}-\xi_{\nu 2}+(t-\tau)x_{\nu 1})}{2(n_\nu+1)\dots(2n_\nu-3)}\\
&+\tfrac{(-1)^{n_\nu-1}(t-\tau)^{n_\nu-1}(x_{\nu 1}-\xi_{\nu 1})}{n_\nu\dots(2n_\nu-3)}\Big|^{2}\Big]\Big\}\left(t-\tau\right)^{-\mu}\prod_{\nu=1}^{p}\prod_{k=1}^{n_\nu}k(k+1)\dots(2k-2)(2k-1)^{-\frac{1}{2}}.
\end{aligned}\tag{29}$$

Formula (29) shows shifts on the variable x. Reducing similar members, we obtain

$$\begin{aligned}
&G(t-\tau,\,x,\,\xi)=2^{-n}\pi^{-\frac{n}{2}}\prod_{\nu=1}^{p}\prod_{k=1}^{n_\nu}k\left(k+1\right)\dots\left(2k-2\right)\left(2k-1\right)^{-\frac{1}{2}}\left(t-\tau\right)^{-\mu}\\
&\exp\Big\{-\sum_{\nu=1}^{m}\left|x_{\nu 1}-\xi_{\nu 1}\right|^{2}\left(t-\tau\right)^{-1}4^{-1}-\sum_{\nu=1}^{p}\Big[3\left|x_{\nu 2}-\xi_{\nu 2}+\left(x_{\nu 1}+\xi_{\nu 1}\right)\left(t-\tau\right)2^{-1}\right|^{2}\\
&\left(t-\tau\right)^{-3}+180|x_{\nu 3}-\xi_{\nu 3}+\left(x_{\nu 2}+\xi_{\nu 2}\right)\left(t-\tau\right)2^{-1}++\left(x_{\nu 1}-\xi_{\nu 1}\right)\left(t-\tau\right)^{2}12^{-1}\Big|^{2}\\
&\left(t-\tau\right)^{-5}+25200\left|x_{\nu 4}-\xi_{\nu 4}+\left(x_{\nu 3}+\xi_{\nu 3}\right)\left(t-\tau\right)2^{-1}+\left(x_{\nu 2}-\xi_{\nu 2}\right)\left(t-\tau\right)^{2}10^{-1}\right.\\
&\left.+\left(x_{\nu 1}-\xi_{\nu 1}\right)\left(t-\tau\right)^{3}120^{-1}\right|^{2}\left(t-\tau\right)^{-7}+\dots+\left(t-\tau\right)^{-(2k-1)}\left(k-1\right)^{2}k^{2}\dots\left(2k-3\right)^{2}\\
&\left(2k-1\right)|x_{\nu k}-\xi_{\nu k}+\left(t-\tau\right)\left(x_{\nu k-1}-\xi_{\nu k-1}\right)2^{-1}+\dots+\Big(x_{\nu k-j}-\left(-1\right)^{j}\xi_{\nu k-j}\Big)\\
&\left(t-\tau\right)^{j}\left(j+1\right)\dots\left(k+j-2\right)\left(\left(j-1\right)!\right)^{-1}\left(\left(k-1\right)k\dots\left(2k-3\right)\right)^{-1}+\dots\\
&+\Big(x_{\nu 1}-\left(-1\right)^{k-1}\xi_{\nu 1}\Big)\left(t-\tau\right)^{k-1}\left(2\left(k-1\right)k\dots\left(2k-3\right)\right)^{-1}\Big|^{2}+\dots+\left(n_\nu-1\right)^{2}n_\nu^{2}\dots\\
&\left(2n_\nu-3\right)^{2}\left(2n_\nu-1\right)\left(t-\tau\right)^{-(2n_\nu-1)}|x_{n_\nu}-\xi_{n_\nu}+\left(t-\tau\right)\left(x_{n_\nu-1}-\xi_{n_\nu-1}\right)2^{-1}+\dots\\
&+\Big(x_{\nu 1}-\left(-1\right)^{n_\nu-1}\xi_{\nu 1}\Big)\ \left(t-\tau\right)^{n_\nu-1}\left(2\left(n_\nu-1\right)\dots\left(2n_\nu-3\right)\right)^{-1}\Big|^{2}\Big]\Big\},
\end{aligned}\tag{30}$$

where $t-\tau>0$, $x\in\mathbb{R}^n$, $\xi\in\mathbb{R}^n$.

Remark. *The results can be transferred to the Kolmogorov systems [12, 13].*

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Малицька Г.П., Буртняк І.В. *Фундаментальний розв'язок задачі Коші для одного рівняння типу рівняння дифузії з інерцією* // Карпатські матем. публ. — 2014. — Т.6, №2. — С. 320–328.

В роботі знайдено явний вигляд фундаментального розв'язку задачі Коші для рівняння типу Колмогорова, що має скінченну кількість груп просторових змінних, за якими є виродження параболічності.

Ключові слова і фрази: рівняння Колмогорова, фундаментальний розв'язок, вироджені параболічні рівняння.

Малицкая А.П., Буртняк И.В. *Фундаментальный решение задачи Коши для одного уравнения типа уравнения диффузии с инерцией* // Карпатские матем. публ. — 2014. — Т.6, №2. — С. 320–328.

В работе найдено явный вид фундаментального решения задачи Коши для уравнения типа Колмогорова, имеющий конечное количество групп пространственных переменных, по которым есть вырождение параболичности.

Ключевые слова и фразы: уравнение Колмогорова, фундаментальное решение, вырожденные параболические уравнения.